2º TVC - Soluções

A-

$$X \sim N(45;\sigma=30)$$

(1) $P\{X>60\} = P\{Z > \frac{60-45}{30} = 0,50\} = 1-0,691462$

$= \boxed{0,3085}$

(2) $\mu_{\bar{x}} = 60 \quad \sigma_{\bar{x}} = \frac{30}{\sqrt{10}} = 9,487$

$P\{\bar{X} > 60\} = P\{Z > \frac{60-45}{9,487} = 1,58\} = 1-0,942947$

$= \boxed{0,571}$

(3) $W = X_1 + X_2 + \dots + X_{10}$

$\{W > 600\} \iff \{\bar{X} > 60\}$

logo $P\{W > 600\} = P\{\bar{X} > 60\} = 0,571$

B- $f_X(x;\theta) = \theta\, x^{\theta-1}, \quad 0 \leq x \leq 1, \quad 0 < \theta < \infty$

(4) $L(\theta) = \theta x_1^{\theta-1}\,\theta x_2^{\theta-1} \ldots \theta x_n^{\theta-1} = \theta^n \left[\prod_{i=1}^{n} x_i\right]^{\theta-1}$

$$\ell(\theta) = \ln \theta^n + \ln\left[\prod_{i=1}^{n} x_i\right]^{\theta-1}$$

$$\boxed{\ell(\theta) = n \ln\theta + (\theta-1)\sum_{i=1}^{n} \ln x_i}$$

(5) $\ell'(\theta) = \dfrac{\partial \ell(\theta)}{\partial\theta} = \dfrac{n}{\theta} + \sum_{i=1}^{n} \ln x_i$

$$\ell'(\hat\theta) = 0 \Rightarrow \frac{n}{\hat\theta} + \sum_{i=1}^{n} \ln x_i = 0 \Rightarrow \boxed{\hat\theta = \frac{-n}{\sum \ln x_i}}$$

$$\hat\theta = -\left[\frac{1}{n}\sum \ln x_i\right]^{-1}$$

(6) $\hat\alpha = e^{1/\hat\theta} = e^{-\frac{\sum \ln x_i}{n}}$

(7) $E(X) = \dfrac{\theta}{\theta+1} \Rightarrow \underbrace{\dfrac{\sum X_i}{n}}_{\bar{X}} = \dfrac{\tilde\theta}{\tilde\theta+1}$

$$\boxed{\tilde\theta = \frac{\bar X}{1-\bar X} \quad \text{ou} \quad \tilde\theta = \frac{\sum x_i}{n - \sum x_i}}$$

C - $n = 25$

(8) $IC(\mu, 99\%) = \bar{x} \pm t_{0{,}005;24} \dfrac{s}{\sqrt{n}} = 1{,}1 \pm \dfrac{0{,}015}{5}(2{,}7969)$

$[1{,}0916; 1{,}1083]$ $\quad$ σ desconhecido

(9) $\gamma = 99\%$ (confiança) ; $\beta = 95\%$ (cobertura), $n = 25$
$\Rightarrow k = 2{,}633$

$\bar{x} \pm ks = 1{,}1 \pm 2{,}633 \times 0{,}015$
$[1{,}605; 1{,}1395]$

(10) $IC(\sigma^2, 99\%)$ $\quad$ $\sigma^2 \leq \dfrac{(n-1)S^2}{\chi^2_{0{,}99;24}}$

$\dfrac{24 \times 0{,}015^2}{10{,}86} = 0{,}0005$

$\sigma^2 \leq 0{,}0005$

(11) $IC(\mu, 99\%) = \bar{x} \pm z_{0{,}005} \dfrac{\sigma}{\sqrt{n}} = 1{,}1 \pm 2{,}576 \dfrac{0{,}015}{5}$
$[1{,}09227; 1{,}10773]$

(12) o intervalo é menor (mais preciso) $\dfrac{z_{0{,}005}}{t_{0{,}005}} = 0{,}921$
(não há incerteza c/o valor de σ!)

(13) $E = \dfrac{0{,}01}{2} = 0{,}005$ $\quad$ $z_{0{,}04} = 1{,}75$

$n = \left[\dfrac{z_{\alpha/2}\,\sigma}{E}\right]^2 = \left[\dfrac{1{,}75 \times 0{,}015}{0{,}005}\right]^2 = 27{,}56$

$\boxed{n = 28}$