# Table of Common Distributions

## Discrete Distributions

### Bernoulli(p)

| pmf | $P(X = x|p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \leq p \leq 1$ |
|-----|------------------------------------------------------------------|
| mean and variance | $EX = p, \quad Var X = p(1 - p)$ |
| mgf | $M_X(t) = (1 - p) + pe^t$ |

### Binomial(n, p)

| pmf | $P(X = x|n,p) = \binom{n}{x}p^x(1-p)^{n-x}; \quad x = 0, 1, 2, \ldots, n; \quad 0 \leq p \leq 1$ |
|-----|------------------------------------------------------------------|
| mean and variance | $EX = np, \quad Var X = np(1 - p)$ |
| mgf | $M_X(t) = [pe^t + (1 - p)]^n$ |
| notes | Related to Binomial Theorem (Theorem 3.2.2). The multinomial distribution (Definition 4.6.2) is a multivariate version of the binomial distribution. |

### Discrete uniform

| pmf | $P(X = x|N) = \frac{1}{N}; \quad x = 1, 2, \ldots, N; \quad N = 1, 2, \ldots$ |
|-----|------------------------------------------------------------------|
| mean and variance | $EX = \frac{N + 1}{2}, \quad Var X = \frac{(N+1)(N-1)}{12}$ |
| mgf | $M_X(t) = \frac{1}{N} \sum_{i=1}^N e^{it}$ |

### Geometric(p)

| pmf | $P(X = x|p) = p(1-p)^{x-1}; \quad x = 1, 2, \ldots; \quad 0 \leq p \leq 1$ |
|-----|------------------------------------------------------------------|
| mean and variance | $EX = \frac{1}{p}, \quad Var X = \frac{1-p}{p^2}$ |
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\( M_X(t) = \frac{e^{xt} \lambda^x}{x!} \), \( t < -\log(1 - p) \)

**Notes**

\( Y = X - 1 \) is negative binomial(1, p). The distribution is memoryless: \( P(X > s | X > t) = P(X > s - t) \).

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**Hypergeometric**

\( \text{pmf} \)

\[ P(X = x | N, M, K) = \binom{M}{x} \binom{N - M}{K - x} / \binom{N}{K}; \quad x = 0, 1, 2, \ldots, K; \quad M - (N - K) \leq x \leq M; \quad N, M, K \geq 0 \]

**Mean and Variance**

\[ EX = \frac{KM}{N}, \quad \text{Var } X = \frac{KM(N-M)(N-K)}{N(N-1)} \]

**Notes**

If \( K \ll M \) and \( N \), the range \( x = 0, 1, 2, \ldots, K \) will be appropriate.

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**Negative binomial(\(r, p\))**

\( \text{pmf} \)

\[ P(X = x | r, p) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1 \]

**Mean and Variance**

\[ EX = \frac{r(1-p)}{p}, \quad \text{Var } X = \frac{r(1-p)}{p^2} \]

**Mgf**

\[ M_X(t) = \left( \frac{e^{rt} - p}{1 - p} \right)^r, \quad t < -\log(1 - p) \]

**Notes**

An alternate form of the pmf is given by \( P(Y = y | r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}, \ y = r, r+1, \ldots \) The random variable \( Y = X + r \). The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.34.)

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**Poisson(\(\lambda\))**

\( \text{pmf} \)

\[ P(X = x | \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \leq \lambda < \infty \]

**Mean and Variance**

\[ EX = \lambda, \quad \text{Var } X = \lambda \]

**Mgf**

\[ M_X(t) = e^{\lambda(e^t - 1)} \]
**Beta(α, β)**

**pdf**  
\[ f(x|α, β) = \frac{1}{B(α, β)} x^{α-1}(1-x)^{β-1}, \quad 0 \leq x \leq 1, \quad α > 0, \quad β > 0 \]

**mean and variance**  
\[ EX = \frac{α}{α+β}, \quad Var X = \frac{αβ}{(α+β)^2(α+β+1)} \]

**mgf**  
\[ M_X(t) = 1 + \sum_{k=1}^{∞} \left( \prod_{r=0}^{k-1} \frac{α+r}{α+β+r} \right) \frac{t^k}{k!} \]

**notes**  
The constant in the beta pdf can be defined in terms of gamma functions, \( B(α, β) = \frac{Γ(α)Γ(β)}{Γ(α+β)} \). Equation (3.2.18) gives a general expression for the moments.

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**Cauchy(θ, σ)**

**pdf**  
\[ f(x|θ, σ) = \frac{1}{πσ} \frac{1}{1+ \left( \frac{x−θ}{σ} \right)^2}, \quad -∞ < x < ∞; \quad -∞ < θ < ∞, \quad σ > 0 \]

**mean and variance**  
do not exist

**mgf**  
does not exist

**notes**  
Special case of Student’s t, when degrees of freedom = 1. Also, if \( X \) and \( Y \) are independent n(0, 1), \( X/Y \) is Cauchy.

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**Chi squared(p)**

**pdf**  
\[ f(x|p) = \frac{1}{Γ(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}, \quad 0 \leq x < ∞; \quad p = 1, 2, ... \]

**mean and variance**  
\[ EX = p, \quad Var X = 2p \]

**mgf**  
\[ M_X(t) = \left( \frac{1}{1-2t} \right)^{p/2}, \quad t < \frac{1}{2} \]

**notes**  
Special case of the gamma distribution.

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**Double exponential(μ, σ)**

**pdf**  
\[ f(x|μ, σ) = \frac{1}{2σ} e^{-|x-μ|/σ}, \quad -∞ < x < ∞, \quad -∞ < μ < ∞, \quad σ > 0 \]

**mean and variance**  
\[ EX = μ, \quad Var X = 2σ^2 \]

**mgf**  
\[ M_X(t) = \frac{e^{μt}}{1-(σt)^2}, \quad |t| < \frac{1}{σ} \]

**notes**  
Also known as the Laplace distribution.
**Exponential(\(\beta\))**

\[
f(x|\beta) = \frac{1}{\beta} e^{-x/\beta}, \quad 0 \leq x < \infty, \quad \beta > 0
\]

**mean and variance**

\[
EX = \beta, \quad Var X = \beta^2
\]

**mgf**

\[
M_X(t) = \frac{1}{1 - \beta t}, \quad t < \frac{1}{\beta}
\]

**notes**

Special case of the gamma distribution. Has the memoryless property. Many special cases: \(Y = X^{1/\gamma}\) is Weibull, \(Y = \sqrt{2X/\beta}\) is Rayleigh, \(Y = \alpha - \gamma \log(X/\beta)\) is Gumbel.

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**Gamma(\(\alpha, \beta\))**

\[
f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad 0 \leq x < \infty, \quad \alpha, \beta > 0
\]

**mean and variance**

\[
EX = \alpha \beta, \quad Var X = \alpha \beta^2
\]

**mgf**

\[
M_X(t) = \left(\frac{1}{1 - \beta t}\right)^\alpha, \quad t < \frac{1}{\beta}
\]

**notes**

Some special cases are exponential (\(\alpha = 1\)) and chi squared (\(\alpha = p/2, \beta = 2\)). If \(\alpha = \frac{3}{2}\), \(Y = \frac{1}{\sqrt{X/3}}\) is Maxwell. \(Y = 1/X\) has the inverted gamma distribution. Can also be related to the Poisson (Example 3.2.1).

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**Logistic(\(\mu, \beta\))**

\[
f(x|\mu, \beta) = \frac{1}{\beta} \frac{e^{-(x-\mu)/\beta}}{1 + e^{-(x-\mu)/\beta}}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0
\]

**mean and variance**

\[
EX = \mu, \quad Var X = \frac{\pi^2}{3} \beta^2
\]
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**Lognormal(μ, σ^2)**

_pdf_  
\[ f(x|μ, σ^2) = \frac{1}{\sqrt{2πσ}} \frac{e^{-(\log x - μ)^2/(2σ^2)}}{x}, \quad 0 \leq x < ∞, \quad -∞ < μ < ∞, \quad σ > 0 \]

_mean and variance_  
\[ EX = e^{μ+(σ^2)/2}, \quad Var X = e^{2(μ+σ^2)} - e^{2μ+σ^2} \]

_moments_ (mgf does not exist)  
\[ EX^n = e^{nμ+n^2σ^2/2} \]

_notes_  
Example 2.3.5 gives another distribution with the same moments.

**Normal(μ, σ^2)**

_pdf_  
\[ f(x|μ, σ^2) = \frac{1}{\sqrt{2πσ}} e^{-(x-μ)^2/(2σ^2)}, \quad -∞ < x < ∞, \quad -∞ < μ < ∞, \quad σ > 0 \]

_mean and variance_  
\[ EX = μ, \quad Var X = σ^2 \]

_mgf_  
\[ MX(t) = e^{μt+σ^2t^2/2} \]

_notes_  
Sometimes called the Gaussian distribution.

**Pareto(α, β)**

_pdf_  
\[ f(x|α, β) = \frac{βα^α}{x^{α+1}}, \quad a < x < ∞, \quad α > 0, \quad β > 0 \]

_mean and variance_  
\[ EX = \frac{βα}{β-1}, \quad β > 1, \quad Var X = \frac{βα^2}{(β-1)^2(β-2)}, \quad β > 2 \]

_mgf_  
does not exist

**t**

_pdf_  
\[ f(x|ν) = \frac{Γ(ν/2)}{Γ(1/2)ν^{1/2}} \frac{1}{\sqrt{ν}} \frac{1}{\left(1+(x^2/ν)\right)^{1/2}}, \quad -∞ < x < ∞, \quad ν = 1, \ldots \]

_mean and variance_  
\[ EX = 0, \quad ν > 1, \quad Var X = \frac{ν}{ν-2}, \quad ν > 2 \]

_moments_ (mgf does not exist)  
\[ EX^n = \frac{Γ(ν+α)}{Γ(1/2)ν^{1/2}} \sqrt{π} \frac{1}{ν-α} ν^{n/2} \text{ if } n < ν \text{ and even,} \]
\[ EX^n = 0 \text{ if } n < ν \text{ and odd.} \]

_notes_  
Related to F (F_{1,ν} = t_ν^2).
### Uniform (a, b)

**pdf** \[ f(x|a, b) = \frac{1}{b-a}, \quad a \leq x \leq b \]

**mean and variance** \[ EX = \frac{a+b}{2}, \quad Var X = \frac{(b-a)^2}{12} \]

**mgf** \[ M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t} \]

**notes** If \( a = 0 \) and \( b = 1 \), this is a special case of the beta (\( \alpha = \beta = 1 \)).

### Weibull (\( \gamma, \beta \))

**pdf** \[ f(x|\gamma, \beta) = \gamma \beta x^{\gamma-1} e^{-x^{\gamma}/\beta}, \quad 0 \leq x \leq \infty, \quad \gamma > 0, \quad \beta > 0 \]

**mean and variance** \[ EX = \beta^{1/\gamma} \Gamma \left( 1 + \frac{1}{\gamma} \right), \quad Var X = \beta^{2/\gamma} \left[ \Gamma \left( 1 + \frac{2}{\gamma} \right) - \Gamma^2 \left( 1 + \frac{1}{\gamma} \right) \right] \]

**moments** \[ EX^n = \beta^n/\gamma \Gamma \left( 1 + \frac{n}{\gamma} \right) \]

**notes** The mgf exists only for \( \gamma \geq 1 \). Its form is not very useful. A special case is exponential (\( \gamma = 1 \)).
Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).