1º TVC - Soluções

A. $\sigma = 0{,}5 \qquad \sigma_{\bar{X}} = \dfrac{0{,}5}{\sqrt{100}} = 0{,}05$

1. $P\{\bar{X}_{100} < 12\} = P\left\{Z < \dfrac{12-12{,}1}{0{,}05}\right\} = \boxed{0{,}022750}$

2. $P\{\bar{X}_n < 12\} = 0{,}01 \Rightarrow z_{0{,}01} = -2{,}33$

$$\frac{12-12{,}1}{\frac{0{,}5}{\sqrt{n}}} = -2{,}33 \Rightarrow \boxed{n = 136}$$

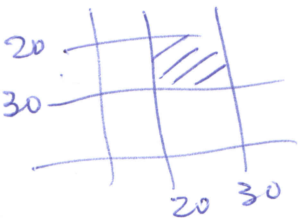
aproxima maior valor

3. $P\{\bar{X}_{100} < 12 \mid \mu = 12{,}1\} = 0{,}01 \qquad z_{0{,}01} = -2{,}33$

$$\frac{12-12{,}1}{\frac{\sigma}{\sqrt{100}}} = -2{,}33 \Rightarrow \boxed{\sigma = 0{,}4292 \text{ onças}}$$

B. $f_{XY}(x,y) = \begin{cases} K(x^2+y^2), & 20 \le x \le 30, \quad 20 \le y \le 30 \end{cases}$

4.



$$\int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dx\,dy = 1 \Rightarrow$$

$$K \times \frac{380.000}{3} = 1 \Rightarrow \boxed{K = \frac{3}{380000}}$$

5.

$$\int_{20}^{26}\int_{20}^{26} \frac{3}{380.000}(x^2+y^2)\,dx\,dy = P\{X<26;\, Y<26\}$$

C-

$X_1 \sim N(0{,}5,\ \sigma = 0{,}1)$ $\rho_{12} = 0{,}7$

$X_2 \sim N(1,\ \sigma = 0{,}2)$

$Y = X_1 + X_2$ (espesor total)

6. $E(Y) = E(X_1 + X_2) = E(X_1) + E(X_2) = \boxed{1{,}5 \text{ mm}}$

$Var(Y) = Var(X_1) + Var(X_2) + 2\,Cov(X_1, X_2)$

$= 0{,}1^2 + 0{,}2^2 + 2 \times 0{,}7 \times 0{,}1 \times 0{,}2$

$= 0{,}0780 \text{ mm}^2$

$\sigma_Y = \sqrt{0{,}0780} = \boxed{0{,}279285 \text{ mm}}$

7. $P\{Y < 1\} = P\left\{Z < \frac{1 - 1{,}5}{0{,}279285}\right\} = \boxed{0{,}036727}$

D-

X_1: nota TVC1 $X_1 \sim N(\mu_1 = 63, \sigma_1 = 8)$

X_2: nota TVC2 $X_2 \sim N(\mu_2 = 60; \sigma_2 = 12)$

$\rho_{12} = 0{,}6$

8. $P\{59 < X_1 < 67\} = P\left\{\frac{59-63}{8} < Z < \frac{67-63}{8}\right\}$

$\boxed{= 0{,}3829}$

9. $P\{X_2 \cancel{>} 60 \mid X_1 = 75\}$

$\mu_{X_2|75} = \mu_2 + \rho \frac{\sigma_2}{\sigma_1}(x_1 - \mu_1) = 60 + (0{,}6)\frac{12}{8}(75-63)$

$= \boxed{70{,}8}$

$\sigma^2_{X_2|75} = \sigma_2^2(1 - \rho^2) = 12^2(1 - 0{,}6^2) = 92{,}16$

$\sigma_{X_2|75} = \sqrt{92{,}16} = \boxed{9{,}6}$

$P\{X_2 > 60 \mid X_1 = 75\} = P\left\{Z > \frac{60 - 70{,}8}{9{,}6}\right\}$

$\boxed{= 0{,}8707}$

E-

X_1: ≤ 2 toques ; $P_1 = 0,70$

X_2: 3 ou 4 toques ; $P_2 = 0,25$

X_3: ≥ 5 toques ; $P_3 = 0,05$

$(X_1, X_2, X_3) \sim$ multinomial

10- $P\{X_1 = 8 ; X_2 = 1 ; X_3 = 1\} = \dfrac{10!}{8!\,1!\,1!} (0,70)^8 (0,25)^1 (0,05)^1$

$\boxed{= 0,064854}$

11. $X_1 + X_2$: 4 ou menos toques

$X_1 + X_2 \sim$ binomial$(10 ; 0,95)$

$\boxed{E(X_1 + X_2)} = 10 \times 0,95 = \boxed{9,5 \text{ chamadas}}$

12- $X_3 | X_1 = 8 \sim$ binomial$\left(2 ; \dfrac{0,05}{0,30} = 0,167\right)$

$\boxed{E(X_3 | X_1 = 8)} = 2 \times 0,167 = \boxed{0,33}$