1º TVC - SOLUCÃO

(A) $\sigma = 0{,}5 \qquad \sigma_{\bar{X}} = \frac{0{,}5}{\sqrt{100}} = 0{,}05$

1) $P\{\bar{X}_{100} < 12\} = P\left\{Z < \frac{12 - 12{,}1}{0{,}05}\right\} = \Phi(-2) = \boxed{0{,}022750}$

2) $P\{\bar{X}_n < 12\} = 0{,}001 \Rightarrow z_{0{,}001} = -2{,}33$

$$\frac{12 - 12{,}1}{\frac{0{,}5}{\sqrt{n}}} = -2{,}33 \Rightarrow n = \left[\frac{(-2{,}33)(0{,}5)}{12 - 12{,}1}\right]^2 = 135{,}7$$

$$\boxed{n = 136}$$

3) $P\{\bar{X}_{100} < 12 \mid \mu = 12{,}1\} = 0{,}01 \Rightarrow z_{0{,}01} = -2{,}33$

$$\frac{12 - 12{,}1}{\frac{\sigma}{\sqrt{100}}} = -2{,}33 \qquad \sigma = \frac{10 \times (12 - 12{,}1)}{-2{,}33}$$

$$\boxed{\sigma = 0{,}4292 \text{ anos}}$$

(B) X: tempo até falhar de lâmina $\quad X \sim \exp(1/3)$

Y: tempo até falhar de manual $\quad Y \sim \exp(1/4)$

4) $P\{\text{torno operar no mínimo 5 anos}\} = P\{X \geq 5;\ Y \geq 5\} \overset{\text{ind.}}{=} P\{X \geq 5\} \cdot P\{Y \geq 5\}$

$$= e^{-5/3}\, e^{-5/4} = (0{,}1888)(0{,}2865)$$

$$= \boxed{0{,}05411}$$

5) $P\{X > t;\ Y > t\} = e^{-t/3}\, e^{-t/4} = 0{,}95 \Rightarrow e^{-\frac{7}{12}t} = 0{,}95$

$$t = \frac{-\ln(0{,}95)}{\frac{7}{12}} = \boxed{0{,}08793 \text{ anos}}$$

(C) $W \sim N(\mu_W = 120 ; \sigma_W = 0{,}5)$ (mm)

$X \sim N(\mu_X = 20 ; \sigma_X = 0{,}1)$ (mm)

$Y \sim N(\mu_Y = 100 ; \sigma_Y = 0{,}4)$ (mm)

V: altura do dispositivo

$V = W + X + Y \sim$ normal

6) $\mu_V = \mu_W + \mu_X + \mu_Y = 120 + 20 + 100 = \boxed{240 \text{ mm}}$

$\sigma_V^2 \overset{\text{ind.}}{=} \sigma_W^2 + \sigma_X^2 + \sigma_Y^2 = 0{,}5^2 + 0{,}1^2 + 0{,}4^2 = \boxed{0{,}42 \text{ mm}^2}$

$\Rightarrow \sigma_V = \sqrt{0{,}42} = \boxed{0{,}64807 \text{ mm}}$

7) $P\{V > 242\} = P\left\{Z > \frac{242-240}{0{,}64807} = 3{,}086\right\} = 1 - \Phi(3{,}09)$

$= 1 - 0{,}998999 = \boxed{0{,}001}$

(D) $\begin{cases} E(X) = 5 \\ E(Y) = 16 \\ E(W) = 20 \end{cases}$ $\begin{cases} Var(X) = 1 \\ Var(Y) = 16 \\ Var(W) = 4 \end{cases}$ $T = W + 2X - 3Y$

8) $E(T) = E(W) + 2E(X) - 3E(Y) = 20 + 2 \times 5 - 3 \times 16 = \boxed{-18}$

9) $Var(T) \overset{\text{ind.}}{=} Var(W) + 2^2 Var(X) + 3^2 Var(Y) = 4 + 4 \times 1 + 9 \times 16 = 152$

$\sigma_T = \sqrt{152} = \boxed{12{,}329}$

10) $P\{T \geq -26\} = P\left\{Z \geq \frac{-26-(-18)}{12{,}329} = -0{,}65\right\} = 1 - \Phi(-0{,}65)$

$= 1 - 0{,}257846 = \boxed{0{,}742154}$

11) $P\{|T - E(T)| > 40\} = 1 - P\{|T - E(T)| \leq 40\} =$

$= 1 - P\{-40 \leq T - E(T) \leq 40\} = 1 - P\left\{\frac{-40}{12{,}329} \leq Z \leq \frac{40}{12{,}329}\right\}$

$= 1 - [\Phi(3{,}24) - \Phi(-3{,}24)] = 1 - [0{,}999402 - 0{,}000598]$

$= \boxed{0{,}0001197}$