TEMPLATE – 1º TVC – 2018-3

(A) $X \sim N(\mu_x = 40, \sigma = 10)$ (peso bagagem classe turística)
$X \sim N(\mu_y = 30, \sigma = 6)$ (peso bagagem classe executiva)

(1)

$$T = X_1 + X_2 + \ldots + X_{50} + Y_1 + Y_2 + \ldots + Y_{12} = \sum_{i=1}^{50} X_i + \sum_{j=1}^{12} Y_j$$

$$\mu_T = \sum_{i=1}^{50} \mu_x + \sum_{i=1}^{12} \mu_y = 50 \times 40 + 12 \times 30 = \boxed{2.360 \text{ lbs}}$$ ✓

supondo independência dos pesos das bagagens.

$$\sigma_T^2 = \sum_{i=1}^{50} \sigma_X^2 + \sum_{i=1}^{12} \sigma_Y^2 = 50 \times 10^2 + 12 \times 6^2 = 5.432$$

$$\sigma_T = \sqrt{5.432} = \boxed{73{,}702 \text{ lbs}}$$ ✓

(2) $$P\{T \leq 2500\} = P\left\{Z \leq \frac{2500 - 2360}{73{,}702} = 1{,}899\right\} = \Phi(1{,}90)$$

$$= \boxed{0{,}971283}$$ ✓

(3) $$\bar{X} \sim N\left(40, \frac{10}{\sqrt{50}}\right)$$

$$\bar{Y} \sim N\left(30, \frac{6}{\sqrt{12}}\right)$$

$$P\{\bar{Y} < \bar{X}\} = P\{\underbrace{\bar{Y} - \bar{X}}_{D} < 0\}$$

$$D = \bar{Y} - \bar{X} \sim N\left(\mu_D = 30 - 40 = -10, \ \sigma_D = \sqrt{\underbrace{\frac{10^2}{50} + \frac{6^2}{12}}_{\sqrt{5}}} = 2{,}236\right)$$

$$P\left\{\bar{Y} - \bar{X} < \frac{0 + 10}{2{,}236} = 4{,}47\right\} \simeq 1$$

(B) $T_i \sim f(t) = \begin{cases} 50\,e^{-25t^2}, & t > 0 \\ 0, & cc. \end{cases}$, $i = 1, 2, \dots, 6$

(4) $R(t) = P\{T > t\} = \int_t^{\infty} 50\,e^{-25x^2}\,dx = -e^{-25x^2}\Big]_t^{\infty}$

$$R(t) = \begin{cases} e^{-25t^2}, & t > 0 \\ 0, & cc \end{cases}$$

(5) $P\{$equipamento funcionar 2 primeiros meses$\} = P\{T_1 > 2\text{ meses}\} \dots P\{T_6 > 2\text{ meses}\}$

$= \left(P\{T_1 > 2\text{ meses}\}\right)^6 = \left[R(2/12)\right]^6 = \left[e^{-25 \times \left(\frac{2}{12}\right)^2}\right]^6$

$= \left[e^{-0{,}6944}\right]^6 = (0{,}49935)^6 = \boxed{0{,}01550}$

(6) $\left(P\{T_1 > t\}\right)^6 = 0{,}05$

$\left(R(t)\right)^6 = 0{,}05 \Rightarrow \left[e^{-25t^2}\right]^6 = 0{,}05$

$\ln\left[e^{-25t^2}\right]^6 = \ln(0{,}05) \Rightarrow 6 \times (-25t^2) = \ln(0{,}05)$

$t = \sqrt{-\frac{\ln(0{,}05)}{6 \times 25}} = \sqrt{0{,}01997} = 0{,}14132$ anos

$\boxed{t = 1{,}70 \text{ meses}}$

$\sqrt[6]{0{,}05} = 0{,}6069$

$W \sim N(\mu_W = 120;\ \sigma_W = 0{,}5)$ (mm)

C. $X \sim N(\mu_X = 20;\ \sigma_X = 0{,}1)$ (mm)

$Y \sim N(\mu_Y = 100;\ \sigma_Y = 0{,}4)$ (mm)

V: altura dispositivo

$V = W + X + Y$

(7) $\mu_V = \mu_W + \mu_X + \mu_Y = 120 + 20 + 100 = \boxed{240 \text{ mm}}$

$\sigma_V^2 \overset{\text{ind.}}{=} \sigma_W^2 + \sigma_X^2 + \sigma_Y^2 = 0{,}5^2 + 0{,}1^2 + 0{,}4^2 = 0{,}42$

$\sigma_V = \sqrt{0{,}42} = \boxed{0{,}64807 \text{ mm}}$

(8) $P\{V > 242\} = P\left\{Z > \dfrac{242-240}{0{,}64807} = 3{,}086\right\} = 1 - \Phi(3{,}09)$

$= 1 - 0{,}998999 = \boxed{0{,}001}$

$E(X) = 5 \qquad Var(X) = 1$

$E(Y) = 16 \qquad Var(Y) = 16$

D- $E(W) = 20 \qquad Var(W) = 4$

$T = W + 2X - 3Y$

(9) $\mu_T = E(T) = E(W) + 2E(X) - 3E(Y) = 20 + 2\times 5 - 3\times 16$

$\boxed{\mu_T = -18}$

(10) $\sigma_T^2 = Var(T) = Var(W) + Var(2X) + Var(3Y)$

$= Var(W) + 2^2 Var(X) + 3^2 Var(Y)$

$= 4 + 4\times 1 + 9\times 16 = 152$

$\sigma_T = \sqrt{152} = 12{,}329$

(11) $P\{|T - E(T)| > 40\} = 1 - P\{|T - E(T)| \leq 40\}$

$= 1 - P\left\{ \frac{-40}{12{,}329} \leq Z \leq \frac{40}{12{,}329} \right\} = 1 - P\{-3{,}24 \leq Z \leq 3{,}24\}$

$= 1 - [\Phi(3{,}24) - \Phi(-3{,}24)] = 1 - [0{,}999402 - 0{,}000598]$

$= 1 - 0{,}998809 = \boxed{0{,}001197}$