A-

$$f(x)=\begin{cases} 3x^2, & 0<x<1 \\ 0, & cc \end{cases}$$

$Y = 1 - X^2 \qquad 0<y<1$

$X = \sqrt{1-Y}$

(1) função de distribuição acumulada de Y ($F_X$)

$F_X(y) = P\{Y \leq y\} = P\{1-X^2 \leq y\} = P\{-X^2 \leq y-1\} = P\{X \geq \sqrt{1-y}\}$

$= P\{X^2 > -(y-1)\} = $

$= \int_{\sqrt{1-y}}^{1} 3x^2\,dx = \frac{\cancel{3}x^3}{\cancel{3}}\Big]_{\sqrt{1-y}}^{1} = 1-(1-y)^{3/2}$

$$F_Y(y)=\begin{cases} 1, & y \geq 1 \\ 1-(1-y)^{3/2}, & 0<y<1 \\ 0, & y \leq 0 \end{cases}$$

(2) Função de densidade de probabilidade de Y

$f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{d}{dy}\left[1-(1-y)^{3/2}\right] = -\frac{3}{2}(1-y)^{1/2}(-1)$

$$f_Y(y)=\begin{cases} \frac{3}{2}(1-y)^{1/2}, & 0<y<1 \\ 0, & cc \end{cases}$$

ou, como $Y = 1-X^2$ é estritamente decrescente p/ $0<x<1$

$f_Y(y) = f_X(x(y))\left|\frac{dx}{dy}\right| = 3\left(\sqrt{1-y}\right)^2 \times \left|\frac{1}{2}(1-y)^{-3/2}\right| = \frac{3}{2}(1-y)^{1/2}$

(3) $P\{0{,}5 \leq Y \leq 2\} = F_Y(2) - F_Y(0{,}5) = 1 - \left[1-(1-0{,}5)^{3/2}\right]$

$= \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} = \boxed{0{,}3536}$

B-

T: tempo entre chamados

$T \sim \exp\left(\lambda = \frac{1}{10}\right)$ $\lambda = 0{,}1$ chamada/min

(4) Probabilidade tempo até 1ª chamada menor que 5 min

$P\{T<5\} = 1 - e^{-\frac{5}{10}} = 1 - e^{-0{,}5} = 1 - 0{,}6065 = 0{,}3935$

(5) $\Delta_t$: Comprimento intervalo de tempo, para

$P\{$pelo menos 1 chamada em $\Delta_t\} = 0{,}9$

X: conta chamados no intervalo $\underline{\Delta_t}$ (em minutos

$X \sim \text{Poisson}(\lambda \cdot \Delta_t)$

$P\{X \geq 1\} = 1 - P\{X=0\}$, mas $P\{X=0\} \Leftrightarrow P\{T > \Delta_t\}$

logo

$P\{T > \Delta t\} = 0{,}10 \implies e^{-\frac{1}{10}\Delta t} = 0{,}10 \implies \Delta t = -\frac{\ln 0{,}10}{0{,}1}$

$\boxed{\Delta t = 23{,}0259 \text{ min}}$

ou pela Poisson

$P\{X=0\} = e^{-\frac{1}{10}\Delta t} \times \frac{\lambda^0}{0!} = 0{,}10 \implies \boxed{\Delta t = 23{,}03 \text{ min}}$

(6) $P\{T<15 \mid T>10\} = P\{T<5\} = 0{,}3935$ [Calculado em (4)]

Questão C

X : comprimento capa de plástico

$$X \sim N(\mu_X = 90^{\underline{2}}\ \text{mm},\ \sigma_X = 0{,}1\ \text{mm})$$

(7) $P(\{X > 90^{\underline{3}}\} \cup \{X < 89^{\underline{7}}\}) = 1 - P\{89^{\underline{7}} \leq X \leq 90^{\underline{3}}\}$

$$= P\left\{\frac{89^{\underline{7}} - 90^{\underline{2}}}{0{,}1} \leq Z \leq \frac{90^{\underline{3}} - 90^{\underline{2}}}{0{,}1}\right\} = \Phi(1) - \underbrace{\Phi(-5)}_{\approx 0}$$

$\approx 0{,}841345$

$P\{\text{peças não conformes}\} = 1 - 0{,}841345 = \boxed{0{,}15866}$

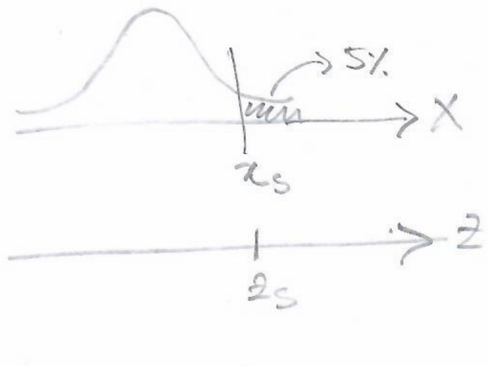
(8) $P\{\text{itens conformes}\} = P\{89^{\underline{7}} \leq X \leq 90^{\underline{3}} \mid \mu = 0{,}90\} =$

$$= P\left\{\frac{89^{\underline{7}} - 90}{0{,}1} \leq Z \leq \frac{90^{\underline{3}} - 90}{0{,}1}\right\} = P\{-3 \leq Z \leq 3\}$$

$$= \Phi(3) - \Phi(-3) = \boxed{0{,}9973}$$

(9) $(0{,}9973)^{10} = 0{,}9733$ (para o processo centrado)

$(0{,}8413)^{10} = 0{,}1776$ (para o processo desajustado)

(10) 


$z_s = \Phi^{-1}(0{,}95) = 1{,}645$

$x_s = \mu_X + \sigma_X z_s =$

$= 90 + 1{,}645 \times 0{,}1$

$\boxed{x_s = 90{,}1645}$ (centrado)

$\boxed{x_s = 90{,}3645}$ (desajustado)

D $\quad M_X(t) = \frac{1}{(1-t^2)}, \quad t<1$

(11) $E(X) = M'_X(0)$

$$M'_X(t) = \frac{-\cancel{(1-t^2)}(-2t)}{(1-t^2)^{\cancel{2}}} = \frac{2t}{(1-t^2)}$$

$$\boxed{E(X) = M'_X(0) = 0}$$

(12) $$M''_X(t) = \frac{2\cancel{(1-t^2)} - 2t\cancel{(1-t^2)}(-2t)}{(1-t^2)^{\cancel{2}}} = \frac{2+4t^2}{(1-t^2)}$$

$$E(X^2) = M''_X(0) = 2$$

$$Var(X) = E(X^2) - [\cancel{E(X)}^{\,0}]^2$$

$$\boxed{Var(X) = 2}$$

(13) $Y = \beta_0 + X$

$$M_Y(t) = e^{\beta_0 t} \cdot M_X(t) = \frac{e^{\beta_0 t}}{[1-t^2]}$$

$$M'_Y(t) = \frac{d}{dt}\left[\frac{e^{\beta_0 t}}{1-t^2}\right] =$$

$$= \frac{\beta_0 e^{\beta_0 t}[1-t^2] - e^{\beta_0 t}(-2t)}{[1-t^2]^2}$$

$$E(Y) = M'_Y(0) = \frac{\beta_0 e^0 [1-0^2] + e^0 (2\beta^2 \cdot 0)}{[1-0^2]^2} = \boxed{\beta_0.}$$